Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International GCE
in Pure Mathematics P2 (WMA12) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⌊ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \ldots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \ldots$$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1 (a)	$h=3$	B1
	Area $\approx \frac{3}{2}\{2+4.81+2\times(3.32+4+4.46)\}$	M1
	$=$ awrt 45.6	A1
		(3)
(b)(i)	States or uses $\log_2 4x^2 = 2\log_2 2x$	M1
	Hence $\int_2^{14} \frac{\log_2 4x^2}{5}\,dx = \frac{2}{5}\times(a) = \text{awrt } 18.2$	A1 ft
(ii)	Attempts to identify $\log_2 2x$ e.g $\log_2\frac{2}{x} = \log_2 4 - \log_2 2x = 2-\log_2 2x$	M1
	Hence $\int_2^{14}\log_2\frac{2}{x}\,dx = \int_2^{14}(2-\log_2 2x)\,dx = 24-45.6 = \text{awrt } -21.6$	A1ft
		(4)
		(7 marks)

(a)

B1 For $h=3$ This is implied by sight of $\frac{3}{2}$ in front of the bracket

M1 For a correct bracket condoning slips (e.g 4.18 instead of 4.81). Look for {first + last + 2(sum of three terms}), but repeated terms scores M0. Must all be y values. Condone "invisible" outside brackets for the M1.

A1 awrt 45.6 An answer of 45.6 can imply the B and M mark if no contrary working is seen and the M may be recovered from "incorrect bracketing" if the answer implies correct bracketing is used..

(b) (i)

M1 States or uses $\log_2 4x^2 = 2\log_2 2x$ (oe)

A1ft For calculating $\frac{2}{5}\times(a) = \text{awrt } 18.2$ but following through on their answer for (a)

(b)(ii)

M1 Attempts to write equation in terms of $\log_2 2x$ using the subtraction rule work to be able to apply (a).

Should be $\log_2\frac{2}{x} = \log_2 a - \log_2 2x\left(= 2-\log_2 2x\right)$ but allow eg. $\log_2\frac{2}{x} = \log_2\frac{2x}{x^2} = \log_2 2x - \log_2 x^2$ for M1.

A1ft For calculating $24-$ their (a) $=$ awrt -21.6 but following through on their answer for (a) and isw if later made positive.

Do not allow for answers that evaluate an integral by calculator (so use of $\log_2 2x - \log_2 x^2$ will likely score M1A0).

Note that trapezium rule used again in (b) is M0 in both parts unless the relevant log work has been seen. Watch out for these as they give the same answers as those in the scheme to 1 dp.

Working must be seen in part (b) to gain credit.

Alt for (b)(ii) $\int_2^{14}\log_2\frac{2}{x}\,dx = \int_2^{14}(\log_2 2 - \log_2 x)\,dx = 12 - \int_2^{14}\log_2 x\,dx$ and $\int_2^{14}\log_2 2x\,dx = \int_2^{14}(\log_2 2+\log_2 x)\,dx = 12+\int_2^{14}\log_2 x\,dx$

solved simultaneously scores the M when the $\log_2 x$ terms are eliminated.

Question Number	Scheme	Marks
2. (a)	For correct term ${}^6C_4 3^2 (ax)^4$	M1
	Sets $15\times 3^2 a^4 = 540$	A1
	$a^4 = 4 \Rightarrow a = \pm\sqrt{2}$	dM1 A1
		(4)
(b)	$\frac{1}{81}\times 3^6 + a^6$	M1 A1
	$=17$	A1
		(3)
		(7 marks)

(a)

M1 For an attempt at the correct term of the binomial expansion which may be embedded within a full expansion.

Accept ${}^6C_4 3^2 (ax)^4$, $\frac{6!}{4!2!}3^2(ax)^4$, $\binom{6}{4}3^2(ax)^4$, $135(ax)^4$ etc but must have correct coefficient and correct powers of 3 and x. Allow equivalents such as ${}^6C_2 3^2 (ax)^4$ and condone a missing bracket for this mark ie. $135ax^4$.

You may condone a slip on the 3 if the rest of an expansion is correct.

A1 For a correct equation in a. Allow $15\times 3^2 a^4 = 540$ or $15\times 3^2 (ax)^4 = 540x^4$ The coefficient must be evaluated for this mark

dM1 For $"135"a^4 = 540 \Rightarrow a = \ldots$ Condone slips on the 540 or errors simplifying an initially correct coefficient, but must be attempting a fourth root. This is implied by a =1.41

A1 $a = \pm\sqrt{2}$ must be exact. Accept $\pm\sqrt[4]{4}$ oe

(b)

M1 For identifying two terms independent of x. Allow if the power of 3 is incorrect but it is clear the correct terms are meant, so $\frac{1}{81}\times$constant term from $(3+ax)^6$ and $a^6x^6\times\frac{1}{x^6} = a^6$ extracted.

A1 Both terms correct and added, ie. $\frac{1}{81}\times 3^6 + a^6$ with a or following through on their value of a but allow if terms are initially correct individually but slips when evaluating.

A1 17 Allow recovery to 17 from use of decimals but 17.0 is A0.

Question Number	Scheme	Marks
3. (a)	Attempts $f\left(-\frac{3}{2}\right)=6\left(-\frac{3}{2}\right)^3+17\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)-12$	M1
	$=0 \Rightarrow (2x+3)$ is a factor *	A1*
		(2)
(b)	$6x^3+17x^2+4x-12=(2x+3)(3x^2+4x-4)$	M1 A1
	$=(2x+3)(3x-2)(x+2)$	dM1 A1
		(4)
(c)	Solves $\tan\theta=-\frac{3}{2}$ or "-2" or "$\frac{2}{3}$"	M1
	$\theta=$ awrt 2.03, 2.16	A1
		(2)
		(8 marks)

(a)

M1 Allow for an attempt at finding a value of $f\left(-\frac{3}{2}\right)=6\left(-\frac{3}{2}\right)^3+17\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)-12$

Sight of embedded values is sufficient. Attempted division is M0

A1* Correctly shows $f\left(-\frac{3}{2}\right)=0$ and states "hence factor" (or substantial equivalent conclusion).

They may state "factor if $f\left(-\frac{3}{2}\right)$=0" in a preamble, in which case accept a minimal conclusion such as //

The M must be scored so there needs to be evidence of either embedded $\left(-\frac{3}{2}\right)$'s or calculations.

(b) Allow for work done in part (a) if referred to in part (b).

M1 Attempt to divide or factorise out $(2x+3)$

By factorisation look for $6x^3+17x^2+4x-12=(2x+3)(3x^2+kx\pm4)$

By division look for
$$\begin{array}{r} 3x^2 \ \pm 4x\text{..........} \\ 2x+3\overline{\big)6x^3+17x^2+4x-12} \\ \underline{6x^3+9x^2} \end{array}$$

By comparing coefficients look for $6x^3+17x^2+4x-12=(2x+3)(ax^2+bx+c)\Rightarrow a=3, c=\pm4, b=...$

A1 Correct quadratic factor $(3x^2+4x-4)$ found. May be within the long division.

dM1 Attempts to factorise their $(3x^2+4x-4)$ usual rules.

A1 $(2x+3)(3x-2)(x+2)$ written out as a product of factors (not as a list) and isw.

Allow $3(2x+3)\left(x-\frac{2}{3}\right)(x+2)$ oe as long as there are three linear factors.

Factors given but no working shown scores M0A0dM0A0

(c)

M1 Solves $\tan\theta = k$ where k is any of their roots to their cubic. Allow if $\tan^{-1}$ seen followed by an answer, or it can be implied by awrt −0.98 or −1.1 or 0.59 or 0.58 truncated (or awrt −56.3 or −63.4 or 33.7 (degrees))

A1 $\theta =$ awrt $2.03, 2.16$ and no other solutions inside the range.

Accept 0.6476π and 0.687π as these give correct answers to 3s.f. but not 0.648π as this is not correct to 3s.f.

Question Number	Scheme	Marks
4	$\int\left(2x^2+7\right)\mathrm{d}x=\frac{2}{3}x^3+7x$ or $\int\left(10-2x^2\right)\mathrm{d}x=10x-\frac{2}{3}x^3$	M1 A1
	Achieves/uses a limit of $\sqrt{5}$	B1
	Area = $17\sqrt{5}-\int_0^{\sqrt{5}}\left(2x^2+7\right)\mathrm{d}x$ | Area = $\int_0^{\sqrt{5}}\left(10-2x^2\right)\mathrm{d}x$	M1
	$=17\sqrt{5}-\frac{2}{3}\times5\sqrt{5}-7\sqrt{5}$ | $=10\sqrt{5}-\frac{2}{3}\times5\sqrt{5}$	M1
	$=\frac{20}{3}\sqrt{5}$	A1
		(6) **(6 marks)**

Line and curve separate

M1 Correct attempt at integration on $2x^2+7$ ie raises a power by one

A1 $\frac{2}{3}x^3+7x$ (need not be simplified)

B1 Achieves a limit of $\sqrt{5}$ either attached to an integral or used to find the area of the rectangle.

M1 For applying Area of R = ±(area of rectangle - area under curve) = $\pm\left(\text{"}17\sqrt{5}\text{"}-\int\left(2x^2+7\right)\mathrm{d}x\right)$

M1 Area under curve = $\left[\frac{2}{3}x^3+7x\right]_0^{\sqrt{5}}=\frac{2}{3}\sqrt{5}^3+7\sqrt{5}$. This is for applying the limits of 0 and their √5 (must be an x value) to the integral which must be a changed function. The application of the 0 may be implied. Can be scored if the rectangle is not considered. May be awarded on decimals correct to 1d.p. (allow if found by calculator (23.1)).

A1 $\frac{20}{3}\sqrt{5}$ or exact (single term) equivalent and can be recovered from a negative made positive

Line and curve together

M1 Attempts ± (line – curve) with correct attempt at integration on their $\pm10\pm2x^2$ (raises a power by one)

A1 $\pm\left(10x-\frac{2}{3}x^3\right)$ (may be either way round)

B1 Achieves a limit of $\sqrt{5}$

M1 For Area = $\pm\int_0^{\sqrt{5}}$ their "line − curve" dx with limits 0 and their √5 which must be an x value.

M1 $\left[10x-\frac{2}{3}x^3\right]_0^{\sqrt{5}}=10\sqrt{5}-\frac{2}{3}\sqrt{5}^3$ This is for applying the limits of 0 (may be implied) and their √5 (must be an x value) to the integral. May be awarded on decimals (1d.p., calculator use ie 14.9 if correct intergral).

A1 $\frac{20}{3}\sqrt{5}$ or exact (single term) equivalent and can be recovered from a negative made positive

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Note that Area of $R=\int_7^{17}\frac{(y-7)^{\frac{1}{2}}}{\sqrt{2}}\mathrm{d}y=\left[\frac{2(y-7)^{\frac{3}{2}}}{3\sqrt{2}}\right]_7^{17}$ is not on specification but allowable and the above scheme can be applied having made x the subject . M1A1 correct integration B1 uses limit 7 M1 attempts integral w.r.t. y with limits their 7 and 17 (need not be correct integration) M1 applies limits A1 answer.

Question Number	Scheme	Marks
5 (a)	Attempts $30000\times r^3=34\,000$	M1
	$r^3=\dfrac{17}{15}\Rightarrow r=1.0426$ Hence $p=4.26$	A1 A1ft
		[3]
(b)	Attempts $30\,000\times(1.0426)^{"N"}=75\,000$ or with > or < etc throughout.	M1
	$(1.0426)^{"N"}=\dfrac{5}{2}$	A1
	Takes logs $"N"=\dfrac{\log\left(\frac{5}{2}\right)}{\log 1.0426}$ (= awrt 21.96)	M1 A1
	$N=22$	B1
		[5] **(8 marks)**

(a)

M1 Attempts to find the common ratio by use of $30000\times r^3=34000$ or $30000\times r^2=34000$. Must involve taking a root. It may be called *p*. Condone slips on the 30000 or 34000.

A1 For awrt 1.04. It may be called *p*

A1ft $p=4.26$ or follow through on a correct percentage to 2 d.p. for their *r* if the method has been earned (usually 6.46). Note it must be 2d.p. for this mark.

(b)

M1 States/uses $30\,000\times("r")^{"N"}=75000$ or $30\,000\times("r")^{"N-1"}=75000$ with their ratio *r,* but not if their percentage is used. E.g. follow through on their 1.0426 but not 4.26

A1 "Correct" intermediate statement $("r")^{"N"}=\dfrac{5}{2}$ or $("r")^{"N-1"}=\dfrac{5}{2}$

Accept $r=\text{awrt }1.04$ or $r=\text{awrt }1.06$ if r^2 was used in (a)

M1 Uses logs correctly $"N"=\dfrac{\log\left(\frac{5}{2}\right)}{\log"1.0426"}$ or $"N-1"=\dfrac{\log\left(\frac{5}{2}\right)}{\log"1.0426"}=$ or $"N"=\log_{"1.0426"}\left(\dfrac{5}{2}\right)=$

Award for the correct solution of any index equation using logs (correctly). The equation may be from incorrect work, e.g having used S_n but must be a solvable equation (ie positive base and operand)

A1 Correct expression for *N* or $N=\text{awrt }21.96$ following correct work

B1 Depends on *r* = awrt 1.04 and ar^N used. It is for 22 or f.t. their "*N*" rounded up to nearest integer providing the only errors have been due to rounding. E.g. For *r* = 1.04 this will be for *N* = 24

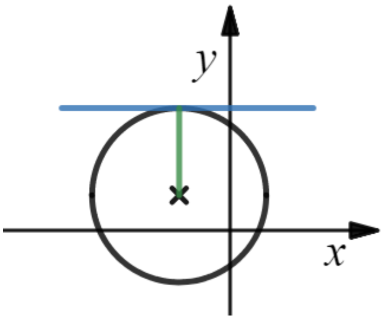
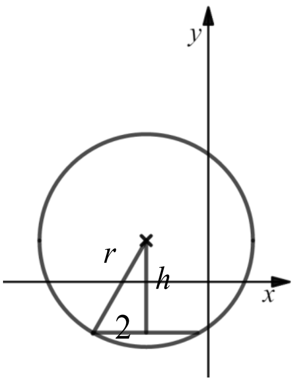
Note: Accept answers in (b) using inequalities and don't be concerned about the inequalities.

S.C. If power *N* – 1 is used then *N* = 22.962 will be reached if all is correct. If they then round down to 22 then the final A and B can be recovered (it is correct work).

SC For trial and improvement M1 Tries $30\,000\times("r")^N$ with *N* either side of 22 or appropriate value for their *r* (eg 15 for 1.0646) with A1 correct values found either side of 75000 for *r* = awrt 1.04 or 1.06

M1 Values for *N* directly either side of 75000 found for their *r*. A1 Must be using *r* = 1.0426. Finds $30\,000\times(1.0426)^{21}=72043$ and $30\,000\times(1.0426)^{22}=75112$.

B1 Deduces 22 or f.t. *N* = 24 for *r* = 1.04.

Question Number	Scheme	Marks
	$x^2+y^2+6x-4y-14=0$	
6.(a)	Attempts $(x\pm3)^2+(y\pm2)^2 .....(=0)$	M1
	(i) Centre $(-3,2)$	A1
	(ii) Radius $\sqrt{27}$ or $3\sqrt{3}$	A1
		(3)
(b)		
	Attempts $y/k="2"\pm"3\sqrt{3}"$	M1
	Both $k=2+3\sqrt{3}$ and $k=2-3\sqrt{3}$	A1
		(2)
	Alt:	
	$y=k\Rightarrow x^2+6x+k^2-4k-14=0$	
	$\Rightarrow b^2-4ac=0\Rightarrow 6^2-4\times1\times\left(k^2-4k-14\right)=0$	
	$\Rightarrow 4k^2-16k-92=0\Rightarrow k=...$	M1
	$k=2\pm3\sqrt{3}$ (oe)	A1
		(2)
(c)		
	Attempts Pythagoras to find h $\quad h^2=r^2-2^2$	M1
	Attempts $p=2-h$	dM1
	$p=2-\sqrt{23}$ (oe)	A1
		(3)
	Alt1: chord touches circle when $x=-1$ (or -5) or need $(x+3)=\pm2$	M1
	$\Rightarrow 2^2+(y-2)^2=27\Rightarrow y=...$ or $(-1)^2+y^2+6(-1)-4y-14=0\Rightarrow y=...$	dM1
	$p=2-\sqrt{23}$	A1
		(3)
	Alt2: Sets $y=p$ and applies difference of roots to be 4	
	$\Rightarrow x^2+6x+p^2-4p-14=0\Rightarrow x=\dfrac{-6\pm\sqrt{36-4(p^2-4p-14)}}{2}$	
	$\Rightarrow 2\sqrt{23+4p-p^2}=4$	M1
	$\Rightarrow p^2-4p-19=0\Rightarrow p=...$	dM1
	$p=2-\sqrt{23}$	A1
		(3)
		(8 marks)

(a)

M1 Attempts to complete the square. Look for $(x\pm3)^2+(y\pm2)^2 .....=0$

M1 may be awarded for a centre of $(\pm3,\pm2)$ (from no, or from incorrect, working)

A1 Centre $(-3,2)$

A1 Radius $\sqrt{27}$ or $3\sqrt{3}$

(b)

M1 Attempts one value of k by $"2"\pm"r"$. In the alternative it is for setting $y = k$ in the circle equation, applying $b^2-4ac=0$ and then solving the resulting quadratic in k, or equivalent method. But substitution of x=0 is M0.

A1 For both $k=2+3\sqrt{3}$ and $k=2-3\sqrt{3}$ Allow $y\leftrightarrow k$ Accept exact equivalents, e.g. with $\sqrt{27}$

Alt by differentiation requires

M1 Achieves $ax+by\frac{\mathrm{d}y}{\mathrm{d}x}+cx+d\frac{\mathrm{d}y}{\mathrm{d}x}=0$ followed by setting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, finding x and substituting into the circle equation then attempting to solve the quadratic. A1 Correct answers.

(c)

M1 For an attempt to use Pythagoras's theorem with 2 and their 'r' to find 'h' or 'h'[2]

dM1 Attempts $p="2"\pm"h"$ Condone $p\leftrightarrow y$ Follow through their y coordinate of the centre.

Can be implied by the correct decimal answer (awrt −2.8)

A1 $p=2-\sqrt{23}$ only Condone $p\leftrightarrow y$ Accept equivalent exact forms e.g $-(\sqrt{23}-2)$

Alternatives are possible:

Alt 1

M1 Identifies an x coordinate where the chord touches the circle, $x = -3 \pm 2$, or applies $x + 3 = \pm2$

dM1 Substitutes into the circle equation (either form) and attempts to solve the quadratic (usual rules)

A1 $p=2-\sqrt{23}$ only Condone $p\leftrightarrow y$

Alt2

M1 Substitutes $y=p$ into C and applies the difference between roots is 4

dM1 Solves the resulting quadratic to find p.

A1 $p=2-\sqrt{23}$ only Condone $p\leftrightarrow y$

Question Number	Scheme	Marks
	$8\tan\theta = 3\cos\theta$	
7 (a)	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta} \rightarrow \quad 8\frac{\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$8\sin\theta = 3\cos^2\theta$ oe	
	Uses $\cos^2\theta = 1-\sin^2\theta \rightarrow \quad 8\sin\theta = 3\left(1-\sin^2\theta\right)$	M1
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ *	A1 *
		(3)
(b)	$(3\sin 2x - 1)(\sin 2x + 3) = 0$	M1
	Critical value(s) of $\frac{1}{3}, (-3)$	A1
	Correct method to find x from their $\sin 2x = \frac{1}{3}$	dM1
	$x = \text{awrt } 9.74°, 80.26°$	A1
		(4)
		(7 marks)

(a)

M1 Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (oe) in $8\tan\theta = 3\cos\theta$. Condone slips in coefficients.

M1 Multiplies by $\cos\theta$ and uses $\cos^2\theta = 1-\sin^2\theta$ (oe) to produce an equation in just $\sin\theta$

A1* Proceeds to $3\sin^2\theta + 8\sin\theta - 3 = 0$ with no arithmetical or notational errors.
No mixed variables within the lines of the "proof"
An example of a notational error is $\cos\theta^2$ for $\cos^2\theta$ (Note that this would not lose the M1)

(b)

M1 Attempts to find the critical values of the given quadratic. Implied by correct values given if no working shown. Allow if just $\frac{1}{3}$ is given.

A1 Critical value of $\frac{1}{3}$ and no incorrect second value. The -3 need not be stated. Accept awrt 0.333 for $\frac{1}{3}$

dM1 A correct method to find one value of x from their $\sin 2x = \frac{1}{3}$
It requires both arcsin and $\div 2$ and may be implied by one answer to one decimal place rounded or truncated eg awrt 9.7 or awrt 80.3 (or for 2sf radian answer awrt 0.17, or awrt 1.4)

A1 $x = \text{awrt } 9.74°, 80.26°$. A0 if extra solutions are given in the range but ignore solutions outside.

Note that M1A0M1A1 is possible if correct answers come from a critical value of $\frac{1}{3}$ if a second incorrect C.V. was also found but does not lead to extra solutions in the range.

Question Number	Scheme	Marks
8 .(i)	States $(S=)\quad a+\quad (a+d)\quad +......................\{a+(n-2)d\}+\{a+(n-1)d\}$	B1
	$(S=)\underline{\{a+(n-1)d\}+\{a+(n-2)d\}+.....................(a+d)}\quad +a$ and adds	M1
	$2S=n\big(2a+(n-1)d\big)\Rightarrow S=\dfrac{n}{2}\{2a+(n-1)d\}\quad *$	A1*
		(3)
(ii)	(a) $u_5=22$	B1
	(b) $\sum_{n=1}^{59}u_n=\big(5+10+15+...\big)+\big(-3+3-3+...\big)$	
	$=\dfrac{59}{2}\{10+58\times5\}+(-3)=8850-3=8847$	M1 B1 A1
		(4)
		(7 marks)

(i)

B1 Writes down an expression for S in a minimum of 3 in a and d terms including the first and last terms. Eg. States that $S=a+\big(a+d\big)+\big(a+2d\big)+...........................+a+(n-2)d+a+(n-1)d$

$S=a+\big(a+d\big)+...+l$ scores B1 only if $l=a+(n-1)d$ is later identified as only two terms in a and d.

M1 Attempts to reverse their sum and add terms. Must include at least two pairs of matching terms to be enough to establish the pattern (allow if second sum misses last terms).

A1* Correctly achieves the given result including the intermediate line $2S=n\{2a+(n-1)d\}$ There must be no errors and at least 3 terms should have been shown for the sum and its reverse.

If $S=a_1+a_2+...+a_n$ is used allow the B and final A only if $a_m=a+(m-1)d$ (oe) is clearly identified in the working, or other clear reasoning why each term gives $2a+(n-1)d$, but the M may be gained.

If commas used instead of + in the summation, eg $S=a,\big(a+d\big)...,a+(n-2)d,a+(n-1)d$ the score B0 as no correct sum, but allow M1A1 if the sum is implied by working and all else is correct.

If you see other attempts you feel are worthy of credit then consult your team leader.

(ii) (a)

B1 22

(ii) (b)

M1 Attempts to use the sum of an AP with $n=59,a=5,d=5$ Also allow $\dfrac{n}{2}\big(a+l\big)=\dfrac{59}{2}\big(5+59\times5\big)$

B1 For $\sum_{n=1}^{59}3\times\big(-1\big)^n=-3$

A1 8847

Listing terms can score 3/3 as it is a correct method and can be marked per main scheme.

Answers with no incorrect working can score 3/3 If the correct answer appears from incorrect working then apply SC M0B1A0.

There are other variations on how to do this part (b)(ii) such as
Alt 1: Splits to odd and even terms:

$$S_{\text{odd}} + S_{\text{even}} = \frac{1}{2}(2+292)\times 30 + \frac{1}{2}(13+293)\times 29 = 4410 + 4437 = 8847$$

M1 separates into the two sequences and applies summation formula to at least one
B1 Correct expression for both summations (must have correct number of terms for each)
A1 8847

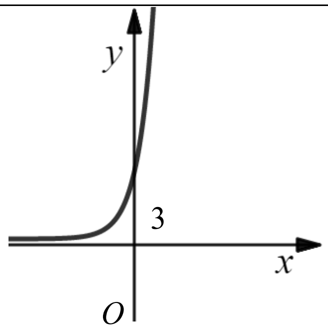
Alt 2: Pairs terms (or can be first term + pairs)

$$S = (2+13)+(12+23)+...+(282+293)+292$$
$$= 15+35+...+575+292 = \frac{29}{2}(15+575)+292 = 8847$$

M1 pairs terms appropriately, may use 1st + (2nd+3rd) + …+ (58th +59th) and applies summation formula to the terms

B1 For the extra term correctly shown 2+… or …+ 292
A1 8847

In general apply
M1 For a correct overall strategy that includes a summation
B1 For dealing with the $\sum(-1)^n$ correctly within the strategy (which may be for a correct overall expression in many cases).
A1 8847

Question Number	Scheme	Marks
9.(a)	 Shape	B1
	$(0,3)$	B1
		(2)
(b)	$6^{1-x} = 3\times 4^x$	
	$(1-x)\log 6 = \log 3 + x\log 4$	M1 dM1 A1
	$x(\log 4 + \log 6) = \log 6 - \log 3$	
	$\Rightarrow x = \dfrac{\log\left(\frac{6}{3}\right)}{\log(4\times 6)} \Rightarrow x = \dfrac{\log 2}{\log 24}$	ddM1 A1*
		(5)
	Alt Method: $6^{1-x} = 3\times 4^x \Rightarrow \dfrac{6}{6^x} = 3\times 4^x$	M1
	$\Rightarrow \dfrac{6}{3} = 6^x 4^x \Rightarrow 2 = 24^x$	dM1A1
	$\Rightarrow \log_{10} 2 = x\log_{10} 24 \Rightarrow \dfrac{\log_{10} 2}{\log_{10} 24}$	ddM1A1*
		(5) **(7 marks)**

(a)

B1 Correct shape and position. Should be increasing, always above the x axis and must be in both quadrants 1 and 2. Be tolerant with the asymptote as long as it does not cross the axis or clearly bend away (but don't be concerned if there is a gap between curve and axis).

B1 Intercept at $(0,3)$ Accept 3 or $(3,0)$ marked on the axis, but $(3,0)$ away from the graph is B0.
Their graph must have crossed the positive y axis to score this mark. Ignore any x intercepts

(b)

M1 Attempt to takes logs (any base, including 6) and attempts either the addition law or the power law
Eg. $\log 6^{1-x} \to (1-x)\log 6$ or $\log 3\times 4^x \to \log 3 + \log 4^x$ (Condone invisible brackets for Ms)

dM1 Takes logs of both sides and attempts **both** the addition law and the power law to achieve a linear equation in x (can still be in any base at this stage)

A1 A correct linear equation in x (any base).

ddM1 Attempts to make x the subject of the formula with no incorrect log work. Allow with just log, but any other base must be changed to base 10 to allow this mark.

A1* Proceeds to the given answer showing a correct intermediate step with no incorrect working seen.

For example $x(\log 4 + \log 6) = \log 6 - \log 3 \Rightarrow x = \dfrac{\log\left(\frac{6}{3}\right)}{\log(4\times 6)}$ or $\ldots \Rightarrow x\log 24 = \log 2 \Rightarrow x = \dfrac{\log 2}{\log 24}$

Alt (b): M1: applies index law to 6^{1-x} ie $6/6^x$
dM1: rearranges to equation of form $a^x = b$ using correct power laws (ie $p^xq^x = (pq)^x$) A1: Correct equation
ddM1: solve equation of the form shown using logs A1: correctly achieved.

Question Number	Scheme	Marks
10.(a)	$y=4x^3-9x+\frac{k}{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=12x^2-9-\frac{k}{x^2}$	M1 A1
	$x=\frac{1}{2},\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 0=12\times\left(\frac{1}{2}\right)^2-9-4k\Rightarrow 4k=-6\Rightarrow k=-\frac{3}{2}$ *	dM1 A1*
		(4)
(b)	Attempts $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=24x-\frac{3}{x^3}$ at $x=\frac{1}{2}$	M1
	$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-12<0$ (Local) Maximum	A1
		(2)
(c)	$12x^2-9+\frac{3}{2x^2}=0\Rightarrow 24x^4-18x^2+3=0$	M1
	$\Rightarrow 3\left(4x^2-1\right)\left(2x^2-1\right)=0$	A1
	$\Rightarrow x^2=\frac{1}{2}\Rightarrow x=\frac{\sqrt{2}}{2}$ oe	dM1 A1
		(4)
		(10 marks)

(a)

M1 Attempts to differentiate reducing any power of x by one

A1 For $12x^2-9-\frac{k}{x^2}$ oe need not see $\frac{\mathrm{d}y}{\mathrm{d}x}$

dM1 Substitutes $x=\frac{1}{2},\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to set up a linear equation in k. (Allow the M if "=0" implied)

A1* Proceeds via $ak+b=0$ or $ak=b$ to achieve $k=-\frac{3}{2}$ following correct work. The "= 0" must have been seen for this mark.

(b) Attempts at first derivative test – send to review if they seem worthy of merit.

M1 Attempts to find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at $x=\frac{1}{2}$. Follow through on their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with one term correct.

A1 Fully correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-12<0\Rightarrow$ (Local) Maximum. Allow with $12-24<0$

(c)

M1 Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and proceeds to a three term quartic equation in x or factorise to give $k\left(ax+\frac{b}{x}\right)\left(cx+\frac{d}{x}\right)=0$ where $kac=24$ and $kbd=3$ (though $k=3$ may be cancelled).

A1 $3\left(4x^2-1\right)\left(2x^2-1\right)=0$ oe. This is implied by $x^2=\frac{1}{2}$ following use of quadratic formula or having identified a quadratic in x^2 e.g. by $y=x^2$ and $24y^2-18y+3\to y=...$

dM1 Solves their $x^2=\frac{1}{2}\Rightarrow x=\pm\frac{\sqrt{2}}{2}$ by taking square root of their positive root

A1 $x=\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ or $\sqrt{\frac{1}{2}}$ but must be positive solution only. Ignore the $x=\frac{1}{2}$ but A0 if others given.

NB Answers directly from the quartic with no further working score M1A0dM0A0

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